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Progress and perspective on the study of the BEC-analog structure in nuclei

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Abstract: The second 0^+ excited state at 7.65 MeV in ^{12}C, known as the Hoyle state, is located near the 3-α breakup threshold and possesses a typical BEC-analog structure. This observation has triggered the intensive theoretical studies of the condensation configuration in nuclear systems, typically represented by the THSR wave function. In the mean time, the experimental investigation of the Hoyle-like states in heavier nuclei has been advanced quite slowly, due mostly to the difficulties in detecting multi-fragments in coincidence and the clarification of the reaction-decay mechanisms. We give here a review of the theoretical and experimental progresses in this field by taking into account the latest experimental outcomes for the 4-α resonances in ^{16}O. Some perspectives are also given towards the possible BEC-like states in neutron-rich systems, which would be of particular importance in exploring the properties of the heavier neutron-rich nuclei and also the neutron stars.

Key words: cluster structure; BEC-analog state; α-conjugate nuclei; neutron-rich nuclei

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1 Introduction

Nucleon clustering is a general phenomenon in nuclear structure configuration, which happens commonly in light nuclei, at the surface of heavy nuclei (α-particle formation and decay) and also frequently in weakly-bound or excited nuclei[1-2]. One famous example is the 0_2^+ (7.65 MeV) state of ^{12}C, also denoted as the "Hoyle state" due to its tremendous significance in the context of carbon synthesis in the universe and hence the origin of life[3]. This state possesses a typical 3-α cluster structure and has attracted intensive experimental and theoretical investigations [1-2,4-9]. Nowadays, it becomes the common understanding that the structure of the Hoyle state is basically featured by the Bose-Eienstein-condensation (BEC), which means that all three α clusters are moving in relative s-wave and in a volume much larger than that of the ground state (g.s.) [5-6,10-12]. It was recongnized that the BEC-like structure can be well described by the Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function [10,13].

Historically, the BEC concept for a finite nucleus has attracted some debates. The main problem concerns the relation between the finite nucleus and the infinite nuclear matter [14-15]. It has been realized that, similar to the successful application of the Bardeen-Cooper-Schripffer (BCS) theory to the finite nuclei, the BEC concept is also meaningful in describing the system composed of a finite number of α-particles, all moving basically in s-wave within a expanded low density volume [6,11,15]. It was demonstrated that the anti-symmetrization between nucleons in different α-clusters can be neglected when the average α-α distance is substantially larger than the α-diameter, and, thus, the localization of the bosons can still be assured [6,15].

Apart from the Hoyle state of ^{12}C, the BEC-analog states have also been proposed for heavier α-conjugate nu-

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clei[8,10,16-18], such as ^{16}O[10,16,19-21], ^{20}Ne[22-24], ^{24}Mg[17,25] and ^{40}Ca[26-27]. However, the corresponding experimental investigations are challenging and no conclusive results were obtained thus far for these heavier systems. The main difficulties lie in the very high excitation of the system, the coincident detection of many decaying α-particles and the clear selection of the targeted reaction-decay channel among many accompanying channels[11].

In this article, we are going to review the major theoretical approaches which give descriptions of the BEC-like states in ^{12}C and in other nuclei. We then outline the experimental progress in finding the evidence for the BEC-like structure, which is still quite limited so far. In the last section, we will make discussion and perspective for the possible experimental work in the near future.

2 Theoretical approaches

The description of the Hoyle-state structure induces a challenging problems for the conventional shell-model approach, within which the 0_2^+ state should appear at an excitation energy much higher than the actually observed 7.65 MeV[6]. On the other hand, the early calculation using the cluster-type orthogonal condition model (OCM) for the α + ^{8}Be(0^+) cluster configuration has correctly reproduced the energy of the Hoyle state and revealed an s-wave coupling between the two subsystems[28]. Since the two α-particles in ^{8}Be(0^+) also interacts via a weakly coupled s-wave, this calculation indicates a weakly coupled 3-α structure in the Hoyle state, resembling a gas-like α condensation[6]. Subsequently, the resonating group method (RGM)[29] and the generator coordinate method (GCM)[30-31] were employed as original microscopic calculations for ^{12}C, which further confirmed the gas-like 3-α structure of the Hoyle state. These approaches had successfully reproduced the basic properties of the Hoyle state, including its excitation energy and the E0- and E2-transition strengths. It would be worth noting that, as early as 1970s [32], studies with 3-α RGM had already reasonably reproduced the excitation energy of the Hoyle state[29-31].

In recent years, the antisymmetrized molecular dynamics (AMD) [33-34] and the fermion molecular dynamics (FMD) [35] models have further confirmed the gas-like clustering structure in the 0_2^+ (7.65 MeV) state of ^{12}C[5]. The AMD model is particularly noteworthy as it gives good description of the nucleon clustering without assuming the preformation of the cluster structure within the nucleus [2]. This advantage relies on using the wave packets, instead of the angular-momentum eigen-states, to represent the nucleon distributions. In addition, the full antisymmetrization, in line with the Pauli exclusion principal, and the orthogonal property between the quantum states also drive the system into the strongly correlated substructures [33]. FDM is essentially the same as the AMD approach, but with a variable width parameter for the gaussian-tyle wave pocket [12]. For the Hoyle state in ^{12}C, both AMD and FDM demonstrate the second 0^+ state in ^{12}C having dominant gas-like 3-α configuration with an extended large size [36-38].

The Algebraic Cluster Model(ACM), originated from the algebraic theory of molecules, is another approach which describes the excited states in ^{12}C quite successfully[9,39]. ACM places the 3-α as an equilateral triangle [39-41]. By employing a general quantization technique based on the Lie algebra $U(\nu+1)$ for systems with ν degrees of freedom, ACM achieves a quantization of the Jacobi variables. Within the ACM framework, the energy levels of some Nα-conjugated nuclei, such as ^{12}C[39], ^{16}O[42] and ^{20}Ne[43], have been calculated.

The idea of the α-condensate in nuclear system was initially proposed by Ropke et al. in their study of the strongly coupled Fermion systems[44]. Later on, the THSR wave function was formulated to present the BEC-analog states in nuclei [10]. It may be regarded as a variation from the Brink wave function [45]. While the Brink wave function place the subsystems (clusters) at random positions inside the nucleus, the THSR wave function requires all subsystems (clusters) moving in s-wave around the center of mass (c.m.) of the whole system [10]. When all the subsystems were bosonic, THSR wave function describes naturally the BEC-analog structure of a nucleus [6,10,12]. Taking the 3-α-clustering system as an example, THSR wave function can be written as[6,12]:

$$\begin{aligned}\Psi_{3\alpha}^{\mathrm{THSR}} =& \mathcal{A}\left\{\exp\left[-\frac{2}{B^2}(\boldsymbol{X}_1^2+\boldsymbol{X}_1^2+\boldsymbol{X}_3^2+)\right]\boldsymbol{\Phi}(3\alpha)\right\}\\ =& \exp(-\frac{6\xi_3^2}{B^2})\mathcal{A}\left[\exp\left(-\frac{4\xi_1^2}{3B^2}-\frac{\xi_2^2}{B^2}\right)\phi_{\alpha_1}\phi_{\alpha_3}\phi_{\alpha_3}\right],\end{aligned} \tag{1}$$

where $\boldsymbol{\Phi}(3\alpha)=\phi_{\alpha_1}\phi_{\alpha_3}\phi_{\alpha_3}$ is the 3-α wave function, $\mathcal{A}$ the antisymmetrization operator and B a parameter characterizing the overall size of the nucleus. $\boldsymbol{X}_i$ is the c.m. of the ith α-cluster. The Jacobi coordinates $\boldsymbol{\xi}$ are related to $\boldsymbol{X}$ according

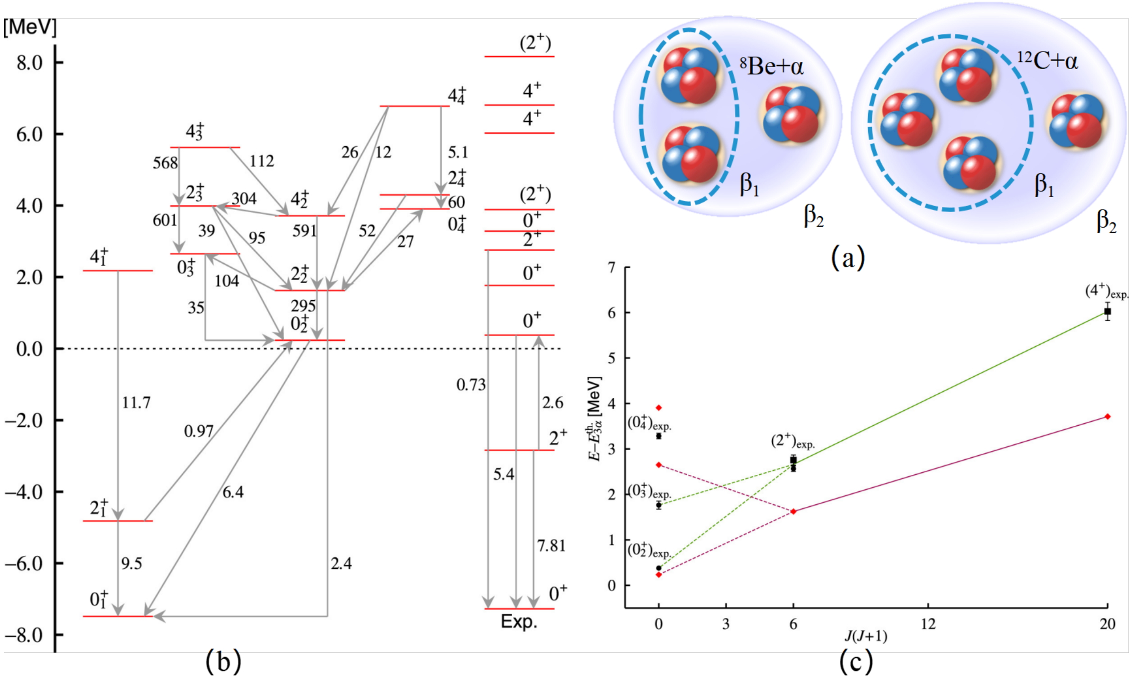

Fig. 1　(Color online) The description of THSR wave function approach. (a) is schematic figure of α+^{8}Be and α+^{12}C cluster structure with "container" picture, respectively. (b) Energy levels [6] and electric transition strengths in ^{12}C between the calculated data with partial available experimental data. (c) the same as (b), but for the comparison of the possible Hoyle rotational band.

to

$$\xi_1 = \boldsymbol{X}_1 - 1/2(\boldsymbol{X}_2 - \boldsymbol{X}_3),$$
$$\xi_2 = \boldsymbol{X}_2 - \boldsymbol{X}_3,$$
$$\xi_3 = \frac{1}{3}(\boldsymbol{X}_1 + \boldsymbol{X}_2 + \boldsymbol{X}_3). \tag{2}$$

The presentation could automatically be applied to the α + ^{8}Be(0_1^+) configuration, in which $\mathcal{A}\exp[\xi_2^2/B^2]\phi_{\alpha_2}\phi_{\alpha_3}$ represents the ^{8}Be(0_1^+) fragment whereas $\exp[4\xi_1^2/3B^2]\phi_{\alpha_1}$ indicates the state of the the remaining α-particle [6,12]. Using the THSR wave function, the Hoyle state in ^{12}C can be correctly reproduced. Furthermore, the THSR wave function has also been employed to explore the BEC-analog states in heavier nuclei, such as in ^{16}O [6,10,12], indicating a general structure feature of the finite nuclear systems at the vicinity of the separation threshold associated with their bosonic subunits. Later on, the THSR wave function was extended to incorporate different size parameter $\boldsymbol{B}$ at different directions (dimensions), allowing to describe the nuclear deformation including the possible linear chain configurations [6,46].

The THSR wave function has become a standard to evaluate the intrinsic BEC-like structure contents in a general quantum state. For instance, the RGM/GCM generated Hoyle state has almost 100% overlap with extended single THSR wave function [6,46-47]:

$$|\langle\phi(3\alpha\mathrm{RGM/GCM})|\phi(3\alpha\mathrm{THSR})\rangle|^2 \approx 100\%, \tag{3}$$

Recently, it has been demonstrated that the THSR wave functions with flexible size parameters can be used to describe various cluster structures, including not only the gas-like but also the non gas-like cluster structures, such as the extremely deformed or asymmetric clustering states [6,11,20,23,48-49]. For instance, the inversion-doublet band (head by $J^\pi = 0^+$ and $J^\pi = 0^-$ states) for the asymmetric ^{16}O + α structure in ^{20}Ne can also be described by a single THSR-type wave function with different size parameters for the core ^{16}O and for the overall ^{20}Ne system, respectively [6,23,49]. This implies that the THSR wave function with flexible size parameters represents the fundamental physical properties of the clustering states. The size parameter is essential here to characterize the cluster-mean-field.

The "containe" model was then proposed, referring to the cluster motion inside the mean-field potential characterized by the size parameter subject to the full antisymmetrization among all nucleons [6,20]. Unlike conventional cluster model wave functions like the Brink wave pocket, the container model describes the relative motion of clusters through the dynamic evolution of the container itself, and the size

of the container evolves from smaller to larger values with increasing excitation energies. The generally nonlocalized wave function can result in localized density distribution for clusters due to the Pauli blocking effect[6,48]. Due to the relatively simple picture and parameter settings, the container model may be regarded as a powerful and promising approach that has the potential to replace the traditional RGM and Brink-GCM models in the description of the cluster dynamics[6,11]. This is particularly true for the intrinsically BEC-like state which overlaps perfectly with the THSR wave functions. As an example, Fig. 1(a) shows schematically the $\alpha + {}^8$Be cluster structure in ^{12}C and the $\alpha + {}^{12}$C cluster structure in ^{16}O within the "container" pictures. Where 2-α or 3-α clusters are confined in a "container" characterized by the parameter β_1, and the remaining α cluster is confined in a larger "container" characterized by β_2 [β_2 is related to the parameter B defined in Eq.(1)]. Fig. 1 (b) and Fig. 1 (c) show the comparison of the Hoyle band in ^{12}C between the experimental data and the theoretical results calculated within the "container" approach. The theoretical results are reasonably agree with the experimental data.

3 Experimental observations

Although the theoretical studies on the BEC-like structure in nuclei have achieved much progresses, the related experimental evidences are still very scarce. Actual observations are mostly concentrated on the Hoyle state of ^{12}C. Ref.[5] has given a summary of measurements for the properties of the Hoyle states, including the excitation energy, decay modes and widths, β and γ transition strengths, inelastic scattering form factor and so on. These measurements are basically indirect and need to be combined with theoretical interpretations in order to reveal the internal structure of the Hoyle state. For instance, the observed inelastic scattering form factor was found to be very sensitive to the size of the final state and provided strong support to an expanded gas-like structure in the Hoyle state[1,11,38,50-52].

One interesting issue would be the possible appearance of the Hoyle state in heavy ion collisions, where many nuclear species can be produced. This has been evidenced once by von Oertzen et al.[11,52], who conducted experiments involving the reaction $^{28}\mathrm{Si} + {}^{24}\mathrm{Mg} \rightarrow {}^{40}\mathrm{Ca} + 3\alpha$. This experiment applied an extensive array of particle detectors to achieve coincident measurements of many charged particles. The γ-ray spectra related to the decay of the heavier fragments, such as ^{40}Ca, ^{39}K and ^{36}Ar, were collected in coincidence with three α-particles randomly emitted to all angles or closely correlated to hit only one small-size detector. The latter case could be interpreted as the appearance of the Hoyle state during the collision. It is interesting to see a strong γ-ray peak corresponding to the excited ^{36}Ar, which is in coincidence with the compact 3-α emission "Hoyle state") but not with the random α emission. The interpretation of this findings is that the 3-α condensation state at the surface of a compound nucleus exhibits a larger diffuseness and radial extension which allows to lowering the Coulomb barrier and, thus, leads to the strongly correlated 3-α emission. To address this kind of phenomenon, it would be advantageous to use the inverse kinematics, which means employing heavier projectiles on lighter targets, allowing to detect the fast-moving decay products, such as multiple α-particles, at forward angles with high efficiency[52]. This detection technology has been implemented in numerous recent experiments[53].

Theoretically, the BEC-like structures have also been predicted for α conjugate nuclei heavier than ^{12}C, such as ^{16}O, ^{20}Ne etc.. Since more particles have to be detected and identified in coincidence, the corresponding experimental work has progressed quite slowly.

For ^{16}O, the sixth 0^+ state at 15.1 MeV, just above the 4-α separation threshold at 14.4 MeV, has been considered as a candidate of the BEC-analog state [20-21,54]. Table 1 summarizes the inclusive measurements for this state. However, the experimental determination of the BEC-like structure must come from the multiple α-decay measurement [55]. The various channels of the 4-α decay from a condensation state in ^{16}O can be expressed as[8]:

$$\begin{aligned} {}^{16}\mathrm{O(BEC)} &\longrightarrow 4\alpha \\ &\longrightarrow {}^{8}\mathrm{Be}(0_1^+) + {}^{8}\mathrm{Be}(0_1^+) \longrightarrow 4\alpha \\ &\longrightarrow {}^{12}\mathrm{C}(0_2^+) + \alpha \longrightarrow 4\alpha \end{aligned} \tag{4}$$

Due to the effect of the Coulomb barrier, the latter channel should be favored[55]. So far, the searches for the α-decay from the ~15.1 MeV state in ^{16}O have been either unsuccessful or characterized by large uncertainties, most likely due to its resonance energy too close to the threshold, resulting in a decay probability as small as the nominal background rate[54,56-58].

The experimental efforts have also been devoted to investigate the Hoyle-analog resonances above 15.1 MeV, considering the possible rotational band members associated

Table 1　Comparison of different experimental data for the 0_6^+ state at 15.1 MeV in ^{16}O

Year	Author	Reaction	Peak Energy/MeV	Width/keV
1972 [59]	T. P. Marvin et al.	^{12}C$(\alpha,\alpha')^{12}$C	15.17(5)	190(30)
1978 [60]	A. D. Frawley et al.	^{15}N(p,α),^{15} N(p,p)	15.10(5)	327(100)
1978 [61]	H. T. Fortune et al.	^{14}N(^{3}He, p)^{16}O	15.103(5)	-
1982 [62]	L. A. Lawrence	^{12}C(α,α'),^{15} N(p,α)	15.066(11)	166(30)
2017 [54]	K. C. W. Li et al.	^{16}O(α,α')	15.076(7)	162(4)
2018 [57]	J. A. Swartz et al.	^{15}N(p,α)	15.1(1)	-
2018 [56]	M. Barbui et al.	^{20}Ne + α	15.2(2)	-

with the BEC-like 0^+ band head. Basically, the experiments might be divided in to two categories. One is resonant scattering between ^{12}C and ^{4}He at center of mass (c.m.) energies covering the interested ^{16}O resonant energies [63-65]. This method suffers from a low energy resolution and a relatively high background, due to the shortcomings such as the inevitable mixing of the non-resonant reaction mechanism and the delimited energy-step size, which often leads to inconsistent results[64-65]. Another category includes the transfer reaction or inelastic excitation followed by 4-α decay from the intermediate ^{16}O resonances[66-68]. This method has the advantage to control the reaction mechanism but the excitation-energy resolution and background contamination depend sensitively on the way to measure the four α particles. Practically, it would be very difficult to measure all 4-α particles with clear particle identification (PID). Previous experiments[66-68] relied on counting the number of hits without PID. Consequently, the background in the excitation energy (E_x) spectra were generally quite high which deteriorated the sensitivity to the obscured resonant states[58,66-68].

Very recently, a new experiment was performed by the Peking University group at the HI-13 tandem accelerator at the China Institute of Atomic Energy, by focusing on the clear PID for all four α-particles decayed from the ^{16}O[69]. The ^{16}O projectile at 96 MeV was inelastically excited by a carbon target. An array of eight charged particle telescopes were used to detect the four α particles. Each telescope consisted of double sided silicon strip detectors with various thicknesses, providing excellent position and energy resolutions. A large number of 4-α events were recorded in coincidence and with full PID. If 3 out of 4 α-particles can be combined to form the Hoyle state of ^{12}C, this event was selected as in the $^{16}\text{O} \rightarrow ^{12}\text{C}(0_2^+, \text{Hoyle state}) + \alpha \rightarrow 4\alpha$ decay channel. The selected events were then used to reconstruct the Hoyle-like states of ^{16}O. For the first time, four narrow resonances just above the 15.1 MeV state were identified from this α + ^{12}C(0_2^+) decay channel with high significance, as demonstrated in Fig.2(lower right panel). Theoretically, there exist some calculations which coincide with these experimental observations. One is the the unified scattering and structure model which predicted a ^{12}C(0_2^+) + α rotational band with 0^+, 2^+, 4^+ and 6^+ members at energies in excellent agreement with the observed resonance peaks [70]. Here the 0^+ member should be a BEC-analog state. Another model, within the field theory superfluid approach, predicted two condensation type 0^+ resonances in the energy range covered by the observed resonances [71]. The above outlined new observation is certainly very encouraging regarding to the existence of the BEC-like state in ^{16}O, similar to what identified in ^{12}C. Further measurement to clarify the spins of the currently observed 4-α resonances in ^{16}O would be of essential importance [69].

There exist also a few experimental studies about the BEC-like states in nuclei heavier than ^{16}O, such as in ^{20}Ne [22,24], ^{24}Mg [25,56] and ^{28}Si [72]. However, the related condensation signatures are still quite ambiguous.

4　Discussion and perspective

As outlined above, the experimental work is still far away from satisfactory for the study of the BEC-analog structure in nuclei[11], in comparison to that of the theory. Apart from the investigations of α conjugate nuclei as instanced above for ^{16}O, we particularly pay attention here to the neutron-rich unstable nuclei which are the actual frontier of nuclear physics studies.

What immediately associated with the well known ^{12}C(Hoyle state) would be the neutron-rich carbon isotopes, such as ^{14,16}C. If there exists the BEC-like resonance in ^{14}C,

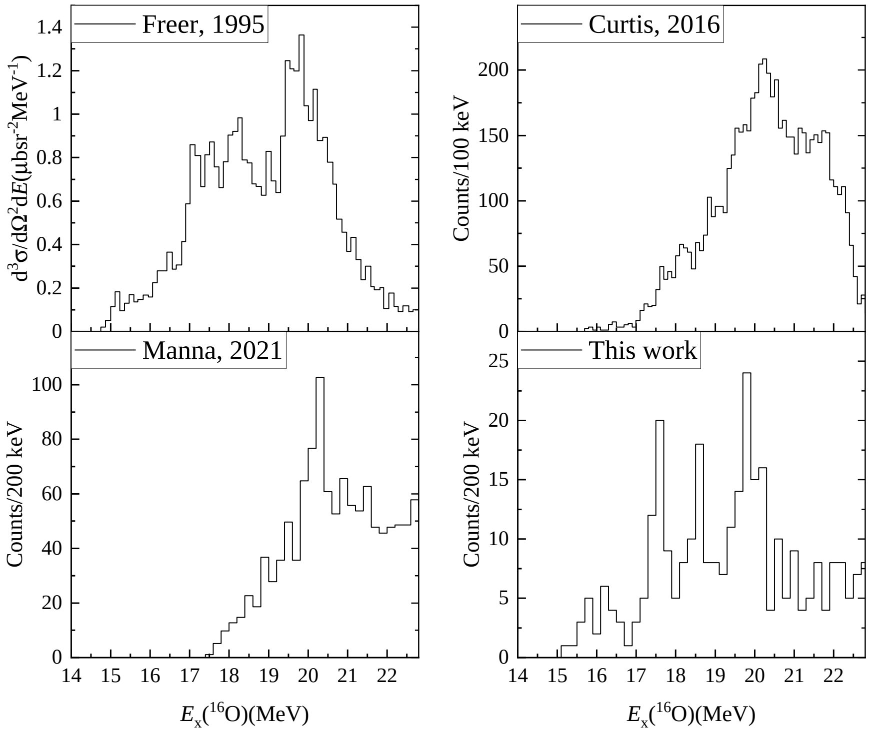

Fig. 2 Comparison of the reported results for ^{16}O-resonances reconstructed from the ^{12}C(0_2^+, Hoyle state) + α channel. Data are from Freer et al.[66], Curtis et al.[68], Manna et al. [58] and Chen et al. (work of our group)[69].

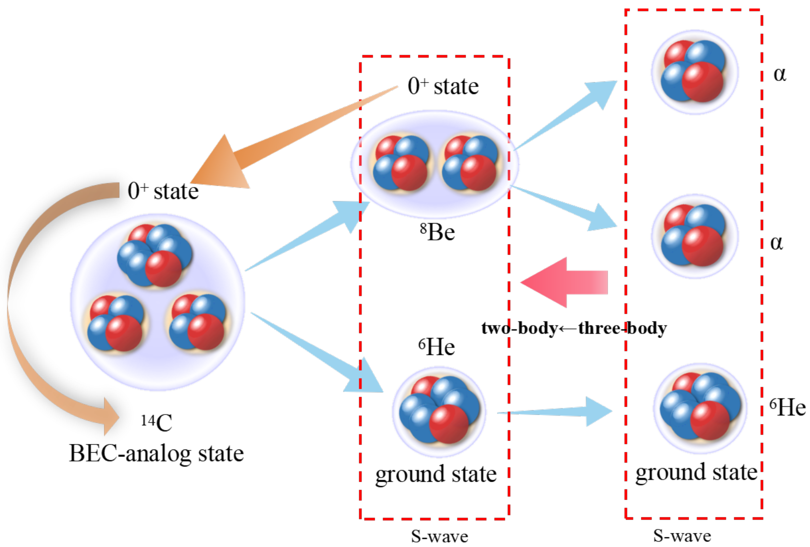

Fig. 3 (Color online) Schematic drawing of the way to search for the BEC-analog resonant state in ^{14}C. The decay processes are denoted by the left-to-right arrows while the reconstruction processes by the right-to-left arrows. Basically, the decay is sequential due to the structural link and the Coulomb barrier effect [55].

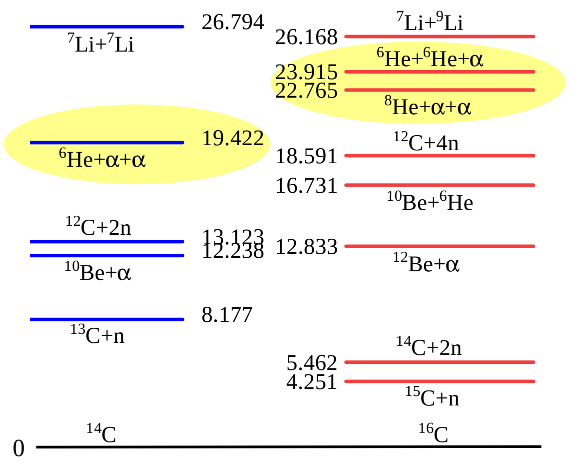

Fig. 4　(Color online) The separation thresholds for different channels from ^{14,16}C (the digits are with a unit of MeV).

the favorite decay channel would be $^{14}\mathrm{C} \to {}^8\mathrm{Be}(0^+) + {}^6\mathrm{He} \to \alpha + \alpha + {}^6\mathrm{He}$, as learnt from ^{12}C and ^{16}O [5,13,69]. Therefore, this kind of experiment could be conducted by measuring the three decay fragments $\alpha+\alpha+{}^6$He, and then selecting the reconstructed $^8\mathrm{Be}(0^+)$ events which can be combined with ^{6}He to form the ^{14}C resonances, as depicted in Fig. 3. Since the ground state of ^{8}Be is already in α-α relative s-wave, the remaining step is to check the total angular momentum of the reconstructed ^{14}C which is the same as the relative orbital angular momentum of the ^{6}He fragment. In order to achieve this measurement, a high excitation energy is required. Fig. 4 illustrates the fragment-separation thresholds for various decay channels from ^{14,16}C, revealing the possible appearance of the BEC-analog states at around 19-24 MeV. In addition, the sufficient event statistics would be essential but diffcult considering the low excitation cross section and the clear detection and identification of the three decay fragments [53,69]. Fortunately, either for ^{14}C or ^{16}C, the three decay helium fragments should all be in their ground states, which simplify the decay paths and thus the Q-value selection [53,69]. Finally, the spin of the mother nucleus, ^{14,16}C, can be determined by the angular correlation analysis or the relative momentum analysis [13,73].

It would be worth emphasizing that the two neutrons in a low density environment may form a compact dineutron pair (^2n) which behaves like a boson (Ref.[74] and references therein). The latest experimental work has achieved to evidence the existence of the $\alpha + {}^2n + {}^2n$ BEC-like configuration in $^8\mathrm{He}(0_2^+)$[75]. This would open the new research direction to search for the BEC-like structure and properties of the low density neutron matter, such as at the surface of the very neutron-rich heavy nuclei or of the neutron star.

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原子核中的类 BEC 结构研究进展与展望

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摘要：
^{12}C 中位于 3-α 破裂阈附近 7.65 MeV 的第二个 0^+ 态 (Hoyle 态)，具有典型的类 BEC 结构，通常可通过 THSR 波函数方法进行有效描述，由此引发了核系统中凝聚结构的深入理论研究。与此同时，较重核中类 Hoyle 态的实验研究却进展缓慢，主要难点在于多个破裂碎片的符合测量以及反应-衰变机制的确定。本文简要回顾了该领域的理论和实验进展，并介绍了最近关于 ^{16}O 中 4-α 共振态的实验结果。此外，也展望了在丰中子系统中存在的类似 BEC 态，这对于探索丰中子重核以及中子星的性质极其重要。

关键词：集团结构；类 BEC；α-共轭核；丰中子核

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